Time-Lapse Marine CSEM Modeling Using Linear and Non-Linear Approximations

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ABSTRACT

One of the electromagnetic induction (EMI) schemes for submarine hydrocarbon reservoir exploration is the control source electromagnetic (CSEM) method. Despite the fact that the marine CSEM method has less accuracy compared to the seismic reflection measurements, it has reduced the cost of modeling and has a relatively high coverage speed. In this research, for 3D and 4D forward modeling of a hydrocarbon reservoir with a regular geometrical shape, the Fredholm integral equation (IE) of second order is used. Whereas for solving full integral equations high performance computers are needed and these computational costs are not affordable, approximations are usually used to solve the electromagnetic problems. The scope of this study is to utilize several approximation methods to solve the integral equation for 4D forward modeling of a regular geometrical shape reservoir using CSEM synthetic data. These approximation methods consist of T-matrix approximation (TMA), Extended Born approximation (EBA), and Born approximation (BA). To numerically verify the performance of the proposed approximations, the inverse modeling of the proposed methods is implemented and then tested in MATLAB. Our results show that the T-Matrix approximation has better accuracy and a wider electrical conductivity application range.

Keywords: MCSEM, Integral equations, Born approximation, Extended Born approximation, T-Matrix approximation

INTRODUCTION

Electromagnetic fields are used in geophysics because of their interactive nature with the medium in which they propagate. This interaction can be utilized to detect certain physical properties of rocks, such as electrical conductivity, dielectric permittivity, and magnetic permeability (Zhdanov 2009). EM methods are based on the study of the propagation of electric currents into the Earth. In hydrocarbon-bearing rocks, resistivity increases profoundly, resulting in a resistivity contrast between the scatterer and the background media. CSEM modeling has become a significant complementary tool for offshore petroleum exploration before drilling. Marine CSEM survey has been used in both 3D modeling and time-lapse water injection into the reservoir. This technique mostly has practical usage in discriminating between the hydrocarbon and the water-filled rocks in addition to estimating the geometry of the hydrocarbon reservoir (Constable 2010).

GREEN’S FUNCTION

Green’s function is an integral kernel that is applied to boundary conditions to solve linear differential equations. The macroscopic dielectric EM field is governed by the Helmholtz equation (Chew 1999). For a homogeneous background, the green’s function form is,

$$G^b(r,r') = \frac{\delta(\mathrm{P})I}{3k^2} + \frac{e^{ik\rho}}{4\pi k^2\rho^3}\left\{\left[1 - ik\rho - (k\rho)^2\right]I - \left[3 - 3ik\rho - (k\rho)^2\right]e_\rho e_\rho\right\} \qquad (1)$$

Where,

$$K \square (-i\mu\sigma\omega)^{\frac{1}{2}} \qquad (2)$$

Where $\mathrm{P} = r - r'$, $\rho = |\mathrm{P}|$, $e_\rho = \dfrac{\mathrm{P}}{\rho}$, δ is Dirac delta function, I is the identity matrix, and μ is magnetic permeability, σ is conductivity, and ω is angular frequency. Due to the singularity, it is important to rewrite green’s function when $r \to r'$ (singularity points) before the EBA and the TMA are formulated. The second term in equation (1) goes up to zero faster than the first term (Chew, 1999). Therefore, the green’s function along the diagonal will be implemented as,

$$G(r,r') = -\frac{\delta(\rho)I}{3k_o^2} = \frac{\delta(\rho)I}{3(\sigma^b + i\omega\varepsilon)} = \frac{1}{3\sigma^b}, \qquad r = r' \qquad (3)$$

INTEGRAL EQUATION FORMULATION

In order to derive the integral equation, green's function technique is usually used. It is assumed that conductivity and electric field are constant in a given cell, and the discretization of the integral equation provides a linear system of equations. In the structure of the IE methods, the conductivity distribution consists of two parts; 1) the background conductivity, σ^b, for the calculation of green's function, and 2) the anomalous conductivity, σ, within the domain of integration (Black and Zhdanov, 2010). In this research, the second order of the Fredholm integral equation is used for the electric field calculation, which is written as follows,

$$E_i(r) = E_i^b(r) + i\omega\mu_0 \int_v G_{ij}(r,r')\Delta\sigma_{jk}(r')E_k(r')dr', \quad (4)$$

Where $E_i^b(r)$ is the background field that can be calculated for a known source $J_i^b(r)$, and the second integral term represents the scattered field. According to Equation (4) the total field is appeared that is the sum of the background field and the scattered field (Habashy et al, 1993).

BORN APPROXIMATION

The Born approximation has wide applicability in solving inverse and forward scattering problems in seismic and electrodynamics methods. BA is developed to avoid solving the super-large system of linear equations for full integral equation algorithms. It works very well only for small conductivity contrasts and low frequencies. This approach considers that the total electric field in the integral terms is approximately equal to the background field, meaning it neglects multiple scattering within the scattered. In other words, the anomalous electric field inside the anomalous domain is zero (Abubakar and Habashy, 2005). Its formulation follows as,

$$\Delta E_i(r) = \int_v G_{ij}^{\ b}(r,r')\Delta\sigma_{jk}(r')E_k(r')dr' \text{ , } E_i(r) \approx E_i^b(r) \quad (5)$$

EXTENDED BORN APPROXIMATION

Habashy et al (1993) presented the Extended-Born technique to improve the Born- approximation. This strategy replaces the total field in the integral equation not by the background field, like in the BA, but considers its projection onto a depolarization tensor, $\Gamma(r')$. Therefore, it is significant to determine the depolarization tensor to allow the replacement of the integral equations for the interior fields by integral representations of these fields. The EBA is based on; 1) considering a homogeneous and isotropic medium permitting the propagation of electromagnetic waves, and 2) recognizing that for interior points, $r \in V$, a dominant contribution to the integral in Equation (4) results from scattering points that are in the neighborhood of the observation point, $\mathrm{r} = r'$, since the green's tensor, $G^b(r,r')$ gives a singularity at that point (Abubakar and Habashy, 2005). Considering Equation (4) the EBA formulation is rewritten as,

$$E_i(r) = E_i^b(r) + \int_v G_{ij}^{\ b}(r,r')\Delta\sigma_{jk}(r')\Gamma_{ki}(r')E_i^b(r')dr' \quad (6)$$

Where $\Gamma(r')$ is the depolarization tensor and $\lambda(r')$ is a tensor that can be written as Equations (7) and (8),

$$\Gamma_{ki}(r') = \left[\mathrm{I} - \lambda_{ik}(r')\right]^{-1} \quad (7)$$

$$\lambda_{ik}(r') = \int_v G_{ij}^{\ b}(r',r'')\Delta\sigma_{jk}(r'')dr'' \quad (8)$$

T-MATRIX APPROXIMATION

The integral equation is approximated by using the T-matrix approach. The computation of the T-matrix is completely independent of the source-receiver configurations, but only needs knowledge about the scattering potential and green's function for the background field. Unlike the BA, the contrasts need not be small. To solve the integral equations, TMA includes all effects of multiple scattering (Jakobsen, 2012). This approximation can be written as Equation (9),

$$T = \Delta\sigma G^b \Delta\sigma + \Delta\sigma G^b \Delta\sigma G^b \Delta\sigma + \ldots = \left[\sum_{k=0}^{\infty} (\Delta\sigma G^b)^k\right]\Delta\sigma \quad (9)$$

TIME-LAPS CSEM

The lateral extent of injected fluid is modeled to monitor small changes in electrical conductivity among increasing pressure inside the reservoir for extraction, while the conductivity of the injected fluid and hydrocarbon remained fixed. This method can detect the changes in contrast of reservoir due to fluid injection in terms of time.

INVERSE THEORY

A regularized least squares method estimates the solution of the inverse problem by finding the model parameters that minimize a particular measure of the length of the estimated data, (Menke 1989). The least squares method uses the $L2$ -norm to quantify the length and can be easily extended to the general linear inverse problems. Tikhonov regularization methods is used to solve discrete ill-posed problems such as Fredholm IEs (Hansen 2010). Least square formulation is as follow,

$$\boldsymbol{m}_{\alpha} = (\boldsymbol{G}^T\boldsymbol{G} + \alpha^2\boldsymbol{I})^{-1}\boldsymbol{G}^T\boldsymbol{d} \qquad (10)$$

Where $\boldsymbol{m}_{\alpha}$ indicates the reservoir conductivity distribution, α is the Tikhonov stabilizer, $\boldsymbol{I}$ is the identity matrix, and $\boldsymbol{d}$ is observed data. Note that the application of the proposed approximation methods leads to a linear inverse problem.

NUMERICAL RESULTS

To verify the validity of these approximation methods a reservoir with a simple geometrical shape was modeled. Information about receivers, source, and reservoir parameters are summarized in table 1.

Table1. receivers, source, and reservoir parameters.

Source and Receivers	Reservoir
Receivers number: 31 at x and 13 at y direction	Thickness (m): 50
Receiver separation (m): 218.5 at x and 538.5 at y direction	Depth (m): 650
Dipole length (m): 100	Background conductivity (S/m): 0.5
Source strength (A): 1000	Reservoir conductivity (S/m): 0.001
Frequency (Hz): 1	Number of grid cells: 56*56*1
Depth (m): 0	Grid volume (m^3) : 25*25*50
Background dimension (m^2) : 7000*7000	Reservoir dimension (m^3) : 1400*1400*50

Figures 1 show forward modeling of the electric field response of simple reservoir by BA, EBA, and TMA, respectively. Referring to Figure 1, the maximum magnitude of the electric field appears at a receiver that is positioned exactly above the reservoir, and as the offset increases, the electric field response becomes weaker. Furthermore, the inversion results of the simulated data using the BA, EBA, and TMA methods are illustrated in figure 2. According to the inverted models, it is evident that the TMA result represents the true model better than BA and EBA.

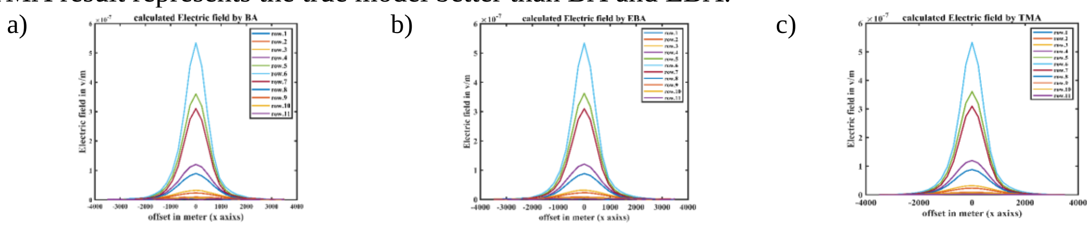

Figure 1. Electric field response recorded by 13 receivers at y direction by, a) BA, b) EBA and, c) TMA

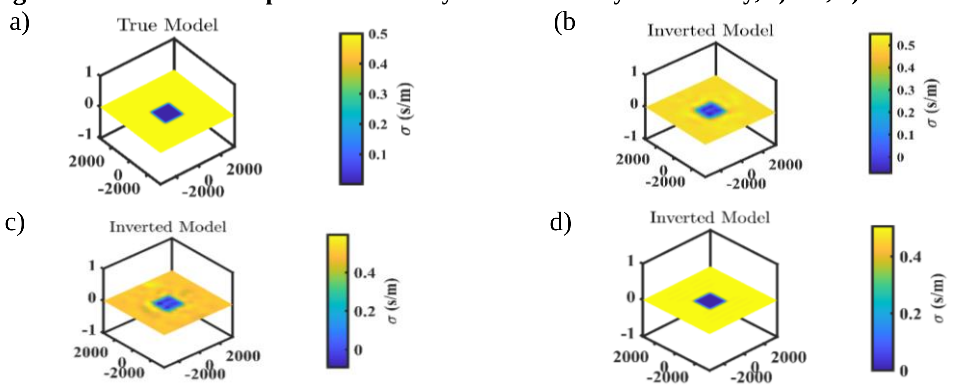

Figure 2. electrical conductivity inverse modeling by a) True model, b) BA (3.85%), c) EBA (3.70%), d) TMA (1.78e-4%).

Figure 3 shows the electric field response of a 750*750*50 m^3 reservoir after water injection to increase pressure in the reservoir by BA, EBA, and TMA, respectively. The electric field magnitude decreases as the water head goes further. The electrical conductivity inverse modelings for three approximations are shown in figure 4. Although there is not a considerable difference between BA and EBA, TMA has an acceptable resolution and relative error with 11.6%.

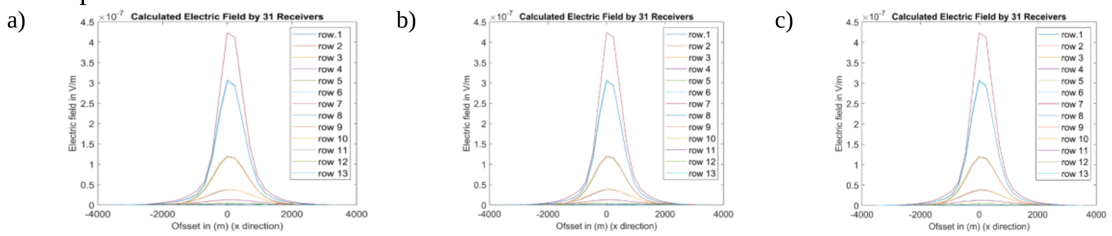

Figure 3. Electric field response recorded by 13 receivers at y direction after 5 years water injection by, a) BA, b) EBA and, c) TMA

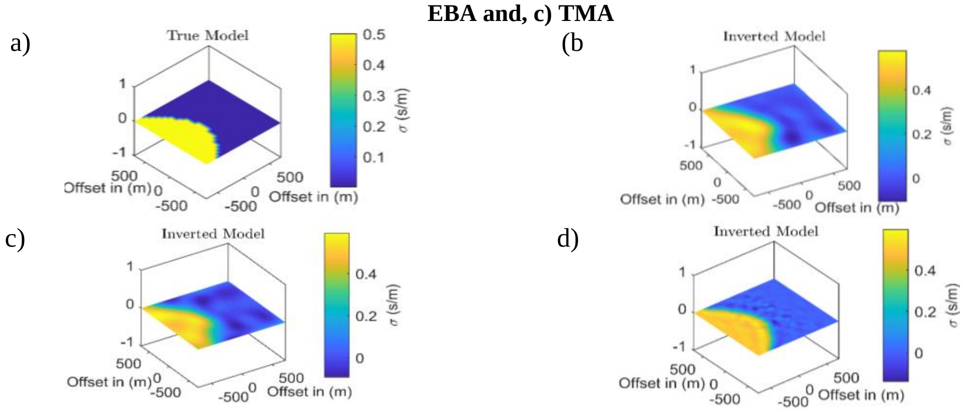

Figure 4. Electrical conductivity inverse modeling after 5 years water injection by, a) BA (re-err: 17.72%), b) EBA (re-err: 16.28%) and, c) TMA (re-err: 11.6%)

CONCLUSION(S)

In this paper, three different approximations (e.g., BA, EBA, and TMA) were applied to the integral equations for MCSEM synthetic data. Considering the numerical experiments, it was demonstrated that the TMA has higher accuracy and a wider range of electrical conductivity application. The TMA roughly estimated the full integral equation solution for EM field, while the EBA is expected to improve the EM field over large conductivity contrasts between the reservoir and background of the assumed model in comparison with the BA. The TMA is valid for high contrasts; therefore, it can be used as a calibrator for BA and EBA. To improve the accuracy, the number of grid cells needs to be increased by considering the computational costs.

REFERENCES

Abubakar, A. and T. M. Habashy., 2005, "A Green function formulation of the Extended Born approximation for three-dimensional electromagnetic modeling", Wave Motion 41(3): 211-227.
Black, N. and M. S. Zhdanov., 2010, "Active Geophysical Monitoring of Hydrocarbon Reservoirs Using EM Methods", 40: 135-159.
Constable, S. (2010). "Ten years of marine CSEM for hydrocarbon exploration." GEOPHYSICS 75(5): 75A67-75A81.
Chew, W. C., 1999, "Waves and Fields in Inhomogeneous Media", IEEE Press: 632.
Habashy, T. M., R. W. Groom, and B. R. Spies., 1993, "Beyond the Born and Rytov approximations: a nonlinear approach to electromagnetic scattering": Journal of Geophysical Research, 98, 1759–1775.
Hansen, P. C. (2010). "Discrete Inverse Problems: Insight and Algorithms." SIAM Journal on Scientific Computing: 53-75.
Menke, W. (1989). "Geophysical Data Analysis: Discrete Inverse Theory." International geophysics Series 45: 1-97
Jakobsen, M., 2012, "T-matrix approach to seismic forward modeling in the acoustic approximation", Stud. Geophys. Geod., 56 (2012), 1−20, DOI: 10.1007/s11200-010-9081-2.
Zhdanov, M.S., 2009," Geophysical electromagnetic theory and methods", (Vol. 43), Elsevier.