Additive versus multiplicative regularization with Sobolev space norm stabilizer: application to FWI

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ABSTRACT

Full-waveform inversion (FWI) is an efficient tool for obtaining high-resolution estimates of the subsurface properties. Existing some errors in the data makes FWI to be ill-posed and the problem needs to be regularized. In additive regularization (AR), the balancing factor needs to be adjusted during the inversion which is more costly for computationally expensive problem such as FWI. While in multiplicative regularization (MR), the data term of objective function, somehow, plays a role of this regularization balancing factor and there is no need to set this factor. In this study, we introduced MR method with Sobolev space norm as a regularization term and used a type of regularization term that has both Tikhonov and total variation regularization functionality. We compared MR with equivalent AR method by application on 2D synthetic noisy datasets. The results show the applicability of MR method for large-scale problems where it can provide more robust solution for the inversion.

Keywords: Full Waveform Inversion, Multiplicative regularization, Sobolev space norm

INTRODUCTION

Seismic full waveform inversion (FWI) has captured researcher's interest in recent years. Its capability in deriving high-resolution models makes it a powerful tool for seismic imaging applications especially in the complex structures. Due to computational challenges, local optimization is preferred to global ones and the popular objective function for FWI is based on least squares of the errors. However, this approach meets problems where data contains noise. Handling this issue leads to introducing regularization term in the objective function. Based on the features inside the model, several regularization methods have been proposed. Among them, Tikhonov regularization has been used more than the others (Asnaashari et al. 2013). Tikhonov regularization is suitable for models with smooth behaviour but its performance is not desirable when the model has some non-smooth features. To cope this problem, researchers proposed alternative methods such as total variation (TV). Based on objective function formulation one can categorize regularization into two categories: additive and multiplicative regularization.

Multiplicative regularization (MR) was introduced to inverse problem society in order to reduce the efforts for determining regularization parameter (Van den Berg et al. 1999; Van den Berg and Abubakar, 2001). A study on theoretical aspects of multiplicative regularization can be found in Gorgin (2015). For FWI, this concept can reduce an extra run-time for the choice of regularization parameter. Hu et al. (2009) implemented the MR method for FWI application and compared MR with different regularization terms.

One of the main issues during FWI is to capture both blocky and smooth features within the model. If the model contains both smooth and blocky features, L_2 and L_1-norm individually cannot satisfy the problem's expectations. Kazei et al. (2017) used Sobolev norm stabilizing functional for FWI problem, which has both TV and Tikhonov functionality. Sobolev norm has been also proposed by Zuberi and Pratt (2016) as a pre-conditioning for FWI which deals with non-linearity of the problem.

In this study, we compared the MR and additive regularization (AR) methods by introducing Sobolev space norm stabilizer into the FWI objective function. Both MR and AR methods were tested on 2D synthetic noisy datasets. The aim of this analysis is to find which type of regularization can deliver more accurate and more robust solution in highly ill-posed inverse problem.

Additive Versus Multiplicative Regularization

In the context of AR, we have the following objective function:

$$\boldsymbol{\Phi}_{\mathrm{AR}} = \boldsymbol{F}(\boldsymbol{m}) + \beta \boldsymbol{R}(\boldsymbol{m}) \quad \textbf{(1)}$$

where $\boldsymbol{F}(\boldsymbol{m})$ denotes the data fidelity term, $\boldsymbol{R}(\boldsymbol{m})$ is the regularization function that ensures either the stability of the problem or constrain the solution toward a priory guesses about the true model. Regardless of different proposed terms as $\boldsymbol{R}(\boldsymbol{m})$, the balancing factor β needs to be calculated during inversion procedure. An alternative approach for reducing the effort for finding β is multiplicative regularization with following objective function (2):

$$\boldsymbol{\Phi}_{\mathrm{MR}} = \boldsymbol{F}(\boldsymbol{m})\boldsymbol{R}(\boldsymbol{m}) \quad (2)$$

In the MR objective function, if we assume $\boldsymbol{m}^{\dagger}$is a minimizer of $\boldsymbol{\Phi}$, then $\nabla_m \boldsymbol{\Phi}(\boldsymbol{m}^{\dagger}) = 0$. We have:

$$\nabla_m \boldsymbol{F}(\boldsymbol{m})\boldsymbol{R}(\boldsymbol{m}) + \nabla_m \boldsymbol{R}(\boldsymbol{m})\boldsymbol{F}(\boldsymbol{m}) = 0 \quad \text{or} \quad \nabla_m \boldsymbol{F}(\boldsymbol{m}) + \left(\frac{\boldsymbol{F}(\boldsymbol{m})}{\boldsymbol{R}(\boldsymbol{m})}\right)\nabla_m \boldsymbol{R}(\boldsymbol{m}) = 0 \qquad \textbf{(3)}$$

Thus the gradient of MR objective function is equivalent to AR if $\beta = \mathbf{F}(\mathbf{m})/\mathbf{R}(\mathbf{m})$. MR can be observed as another version of AR, in which the regularization parameter is within the objective function, inherently.

Multiplicative Regularization With Sobolev Norm

Kazei et al. (2017) introduced $\mathbf{W}_p^1$ norm-based regularization in order to keep the balance between lower and higher wavenumbers of the model in the early iterations. This approach can be also thought as blending criteria between Tikhonov and TV regularization. The expression for $\mathbf{W}_p^1$ norm of the model is:

$$\mathbf{R}_{\mathrm{W}_p^1}(\boldsymbol{m}) = \textstyle\sum_i (\nabla m_i . \nabla m_i + \varepsilon)^{p/2} \qquad \textbf{(4)}$$

where ε is a positive number that guaranties the differentiability and smoothness of $\mathbf{R}_{\mathrm{W}_p^1}(\boldsymbol{m})$ and p determines the functionality of the regularization term. At $p = 2$, performs as Tikhonov and at $p = 1$ it becomes TV. In this study, we compare the performance of MR and AR approaches with Sobolev space norm stabilizer as the model regularization term.

In AR, the value for regularization parameter (i.e. β) was chosen based on the following line search strategy. We start with an enough large β value to initialize the inversion, then we determine β_n (n is the iteration number) according to $\beta_n = \max(\gamma\beta_{n-1}, \beta^{\dagger})$ criteria, in which $\beta^{\dagger}$is the minimizer of data fidelity term in the objective function and γ is a positive constant between zero and one. In the current research, we set the value for ε based on the approach proposed method by Hu et al. (2009).

$$\boldsymbol{\varepsilon} = \frac{\mathbf{1}}{\boldsymbol{N_{gpfreq}N_{data}}}\left(\frac{\boldsymbol{F(m)}}{\boldsymbol{\Delta x \Delta z}}\right)$$

where $\boldsymbol{N_{gpfreq}}$, denotes frequency group number in the inversion, $\boldsymbol{N_{data}}$ is the number of data samples, Δx and Δy are grid size in *x* and *z* directions, respectively. The final value that needs to be set is *p*. its value determines the behaviour of the regularization term. Based on the desired features inside the model one can set this value. In this study, parameter was selected in a logarithmic decreasing manner from 2 to 1. This can help to capture both blocky and smooth features in a multi-scale strategy procedure.

Synthetic Examples

We examined the performance of MR versus equivalent AR approach for two kinds of noisy dataset. The velocity model is a part of Marmousi model (Figure1a). The synthetic data were generated by forward modelling with 80 sources and 159 receivers on the surface, then 3% and 7% random noise added to the data. The smoothed version of the true velocity model was used as the initial model for inversion (Figure 1b). Six frequency groups from 3 to 35 Hz was used in the inversion. Each group has two frequency components. In both MR and AR approaches, we performed 25 iterations of L-BFGS algorithm per each frequency group. The parameters of each objective function were set based on descriptions provided in the previous section. We compared the performance of MR versus AR for both low and high frequencies when data is contaminated with different noise levels. Figure 2 show the result for 3% noise case. As we see in Figure 2, the results of AR and MR methods are pretty similar. Both AR and MR methods have more similarities in low-frequencies. In the final resolved model, we can see some differences between two approaches especially in higher depths. We keep on the examination with higher noise level in the data. Figures 3 results for 7% noise level. In comparison with 3% noise level case, two approaches have differences. However, they still show similar behaviour. For detailed comparison, extracted depth profile at $x = 1680m$ is shown in Figure 4.

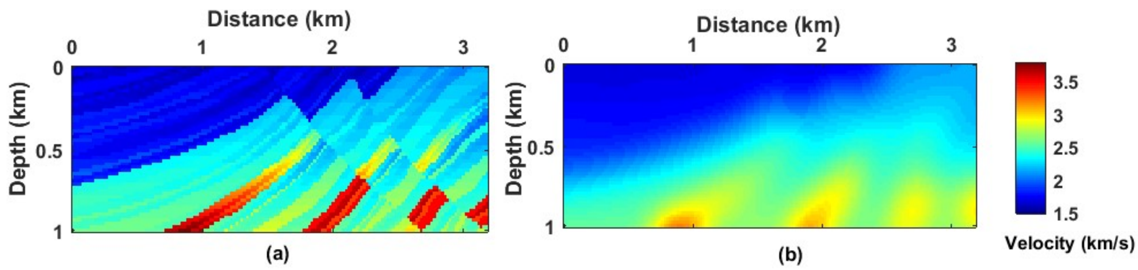

Figure 1 *True velocity model (a); initial smooth model (b)*

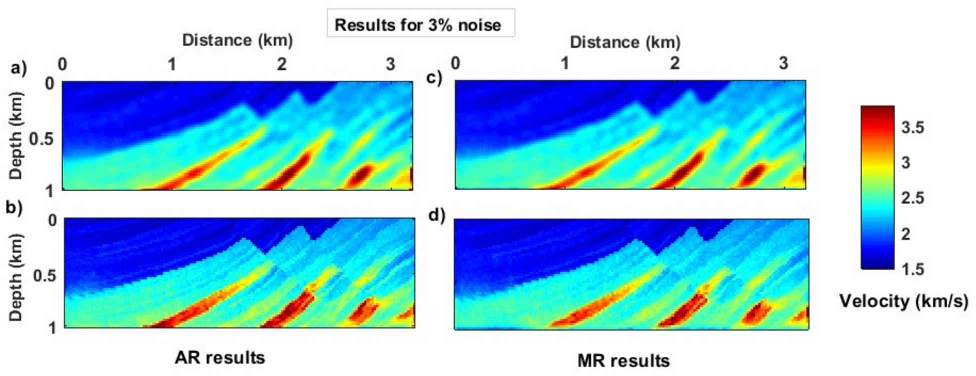

Figure 2 Inversion results for the low-frequency group (top) and final resolved model (bottom) for 3% noisy data. The results for AR approach (a) and (b) are pretty similar for MR (c) and (d).

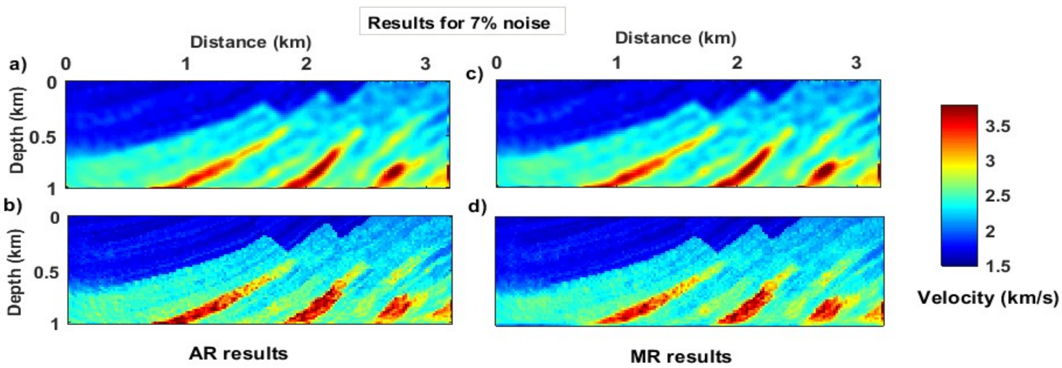

Figure 3 Inversion results for the low-frequency group (top) and final resolved model (bottom) for 7% noisy data. The results for AR approach (a) and (b) are pretty similar for MR (c) and (d).

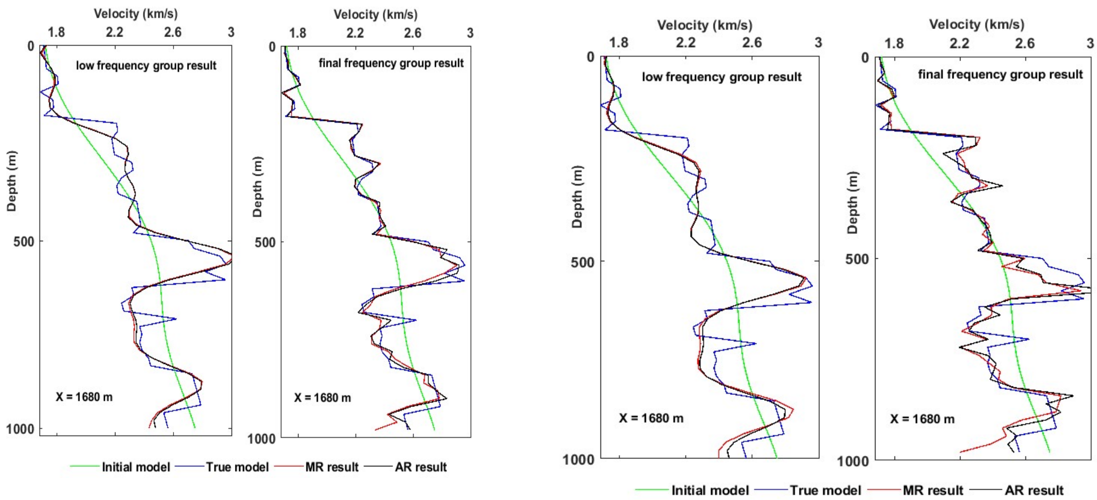

***Figure 4** Detailed comparison of the result for 3% (left) and 7% noisy case, extracted depth profile at* $x = 1680m$ *. The result for MR is comparable to AR.*

Discussion & Conclusions

MR objective function with Sobolev space norm stabilizer was introduced as an alternative approach to deal with large-scale inverse problems especially FWI. In this method, we do not need to set regularization parameter during the inversion, as it is the case for AR function. In this way we can avoid inappropriate selection of the regularization parameter and save extra runtime for selecting it. In MR case, by minimizing the objective function through the iterations and reducing the error in data term, the balancing regularization factor gradually decreases itself and makes an appropriate balance between the data term and the regularization term of the objective function. We compared MR with AR approaches performance on synthetic model contaminated with two different level of noise. Based on the results, the performance of MR is satisfactory comparable with AR. Thus, for large-scale problems, we can propose MR as a more efficient approach for the FWI. In addition, Sobolev norm stabilizer is compatible with the structure of multi-scale strategy.

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