Estimation of b-value without levels of completeness (M_C)

Soheil Azizi[1], Noorbakhsh Mirzaei[2], Javad Kazemian[3]

[1]*M.Sc, Institute of Geophysics, University of Tehran, Tehran, Iran, soheil.azizi@ut.ac.ir*
[2]*Professor, Institute of Geophysics, University of Tehran, Tehran, Iran, nmirzaii@ut.ac.ir*
[3]*Assistant Professor, Department of Earth Sciences, Western University, London, Canada, jkazemia@alumni.uwo.ca*

ABSTRACT

One of the challenges of estimating the Gutenberg–Richter b-value is to determine the levels of completeness of earthquakes (M_C) with numerous methods, and their difficulties. In this research we provided a de-clustered and homogeneous earthquake catalog to estimate the b-value for five major seismotectonic provinces of Iran using a procedure without assumptions on levels of completeness. Comparison of the results of this study with results from previous methods that required determination of M_C indicate the success of the method.

Keywords: Seismicity parameters, Level of Completeness (M_C), b-value, Seismotectonic provinces, Maximum likelihood, Iran.

INTRODUCTION

According to the Gutenberg–Richter frequency–magnitude relation (Gutenberg and Richter, 1944), the number of earthquakes N, having a magnitude equal to or larger than m, can be expressed by the following equation:

$$\log N = a - bm \tag{1}$$

Where parameter a is a measure of the level of seismicity and parameter b describes the ratio of small and large earthquakes. Both parameters have been used in a variety of seismological studies, especially in seismicity simulation (Ogata and Zhuang, 2006), earthquake prediction (Kagan and Knopoff, 1987), and seismic hazard and risk assessment (Cornell, 1968). The b-value can be calculated in various ways (Marzocchi and Sandri, 2003). Maximum likelihood estimation procedure proposed by Aki (1965), which considered earthquake magnitude as a continuous random variable, is still the preferred estimator (Kijko and Smit, 2012). Aki (1965) derived the maximum likelihood estimate of the b-value, as:

$$\beta = \frac{1}{\bar{m} - m_{\min}} \tag{2}$$

Where $\beta = b\ln 10$, The parameters $\bar{m}$ and $m_{\min}$ are the average magnitude and the level of completeness of a given sample of earthquake magnitudes, respectively. Equation 2 requires a complete earthquake catalog that start at a specified level of completeness $m_{\min}$. The utilization of such incomplete catalogs has been discussed by Molchanet al. (1970), Kijko and Sellevoll (1989, 1992), Rosenblueth (1986) and Kijko and Smit (2012), among others, but the most elegant, straightforward, and best-known is the procedure derived by Weichert (1980).

The weakness of previous b-value estimators is its heavy dependence on the M_C. In order to get rid of this dependence Kijko and Smit (2017) proposed a method that does not need to determine the M_C. They proposed two new estimators for estimation of the b-value that are not dependent on the M_C; the method of moments (MM) and the maximum-likelihood (ML).

Methodology

Kijko and Smit (2017) used the apparent PDF defined by multiplying the true PDF of the earthquake magnitude with the probability of completeness $p_c(m)$ and a normalizing coefficient k such that;

$$f_A(m) = kp_c(m)f_M(m), \tag{3}$$

Where $f_A(m)$ is the apparent PDF, $f_M(m)$ is the 'true' PDF of earthquake magnitude, k is a normalizing coefficient that equals:

$$k = \frac{1}{\int_{m_0}^{+\infty} p_c(m)f_M(m)dm}. \tag{4}$$

andis described $f_M(m)$ the catalog. Let us assume that is the smallest observed magnitude in m_0 by the Gutenberg-Richter relation of the following form (Aki, 1965);

$$f_M(m) = \beta \exp\left[-\beta(m - m_c)\right] \tag{5}$$

A schematic illustration of the apparent frequency–magnitude distribution $f_A(m)$ and the theoretical Gutenberg–Richter frequency–magnitude is provided in Figure 1.

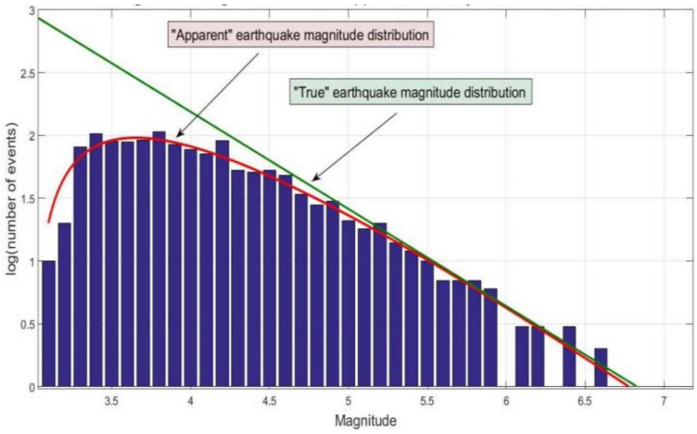

Figure 1. Schematic illustration of the apparent frequency-magnitude distribution (Equation 3) and the theoretical Gutenberg-Richter frequency-magnitude relation (Equation 5) (Kijko and Smit, 2017).

If it is assumed that the Gutenberg–Richter law is correct, then the probability of completeness $p_c(m)$ describes the possible departure of the apparent (observed) magnitude distribution from the exponential nature of the Gutenberg-Richter relation (Equation 5) (Kijko and Smit, 2017). Based on Equations (3), (4), and (5), after normalization, the density distribution of the apparent magnitude can be written as a three-parameter gamma distribution.

$$f_A(x) = \frac{\left\{\beta^{\alpha}(x-\gamma)^{\alpha-1}\exp\left[-\beta(x-\delta)\right]\right\}}{\Gamma(\alpha)}, \ (\alpha > 0;\ \beta > 0;\ x > 0). \tag{6}$$

where $\alpha > 0$, $\beta > 0$, $m > 0$, $\Gamma(\alpha)$ denotes the gamma function, and $x = m - m_{\text{min}}$.

Some possible shapes of the $p_c(m)$ are shown in Figure 2. The proposed $p_c(m)$ does not impose too many limitations and can fit a large variety of shapes (Kijko and Smit, 2017).

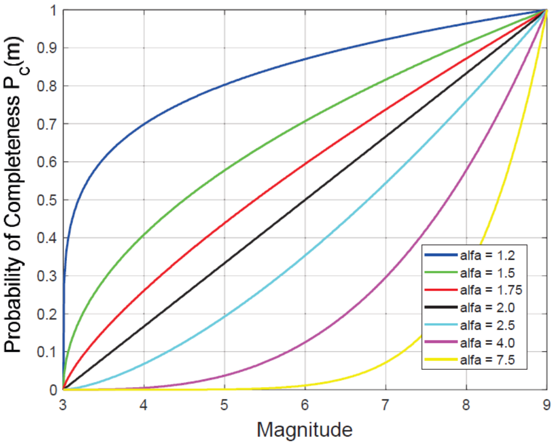

Figure 2. The $p_c(m)$ can take on various shapes that depends on the α (Kijko and Smit, 2017).

Figure 2 shows that the $p_c(m)$ increases with increasing magnitude of earthquakes. $p_c(m)$ for α = 2, is a straight line. The function converges faster for α < 2. Highly incomplete and uncertain catalogs are characterized by α > 2.

Data and results

The study area bounds in 22°-42° N and 42°-66° E (Figure 3). We have provided a declustered and homogeneous earthquake catalog that include historical (pre-1900) and instrumental (1900-2017) earthquakes. The study area has been subdivided into five major seismotectonic provinces by Mirzaei et al. (1998), named: Zagros, Alborz Azarbayejan, Central–East Iran, Kopeh Dagh and Makran.

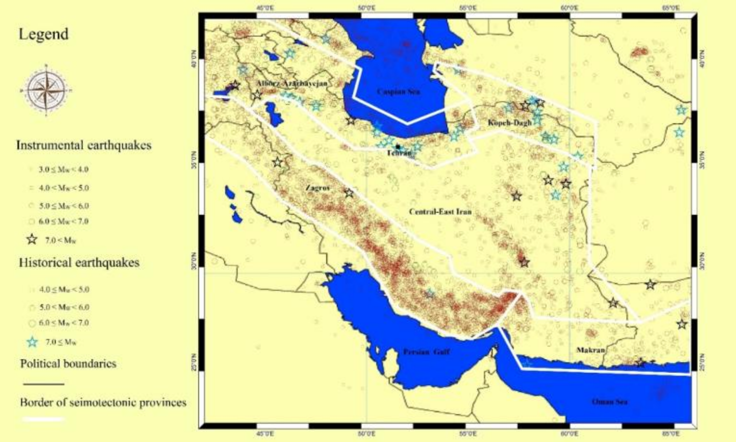

Figure 3. The study area, seismotectonic provinces and distribution of historical (pre-1900) and instrumental (1900-2017) earthquake epicenters. The large earthquakes with magnitudes 7 ≤ M$_W$ are shown with stars.

Time span and frequency of earthquakes in seismotectonic provinces is presented in Table 1. These information provided input data for estimating the b-values by using Kijko and Smit (2017) method. In Table 2, the Gutenberg–Richter b-values in major seismotectonic provinces determined in this study by applying MM and ML methods are shown. Our results have been compared with two previous works by Mirzaei et al. (1997) and Mousavi-Bafrouei et al. (2014) as presented in Table 2.

Table 1. Summary of the input data for estimation of b-value in major seismotectonic provinces of Iran.

Seismotectonic provinces		Alborz-Azarbayejan	Central-East Iran	Kopeh Dagh	Makran	Zagros
Time span	Start	743	734	765	1483	840
	End	2017	2017	2017	2017	2017
Number of earthquakes		1342	1162	450	418	3732

Table 2. Estimation of Gutenberg–Richter b-value from recorded earthquakes in major seismotectonic provinces of Iran by using the method of Kijko and Smit (2017) and comparison of the results with pervious researches by Mirzaei et al. (1997) and Mousavi-Bafrouei et al. (2014).

Seismotectonic provinces		Alborz-Azarbayejan	Central-East Iran	Kopeh Dagh	Makran	Zagros
This Study (Without M_C)	MM method	0.84 +- 0.02	0.85 +- 0.03	0.71 +- 0.04	0.76 +- 0.04	1.09 +- 0.02
	ML method	0.94 +- 0.00	1.03 +- 0.00	0.85 +- 0.00	0.91 +- 0.00	1.24 +- 0.00
Mirzaei et al. (1997)	Hard bound	0.83 +- 0.04	0.84 +- 0.06	0.66 +- 0.08	0.76 +- 0.16	1.04 +- 0.05
	Soft bound	0.86 +- 0.07	0.86 +- 0.09	0.68 +- 0.12	0.76 +- 0.16	1.07 +- 0.06
Mousavi-Bafrouei et al. (2014)	Hard bound	0.91 +- 0.01	0.93 +- 0.02	0.88 +- 0.02	0.85 +- 0.04	1.08 +- 0.01
	Soft bound	0.96 +- 0.03	0.98 +- 0.03	0.91 +- 0.04	0.86 +- 0.05	1.12 +- 0.01

CONCLUSION

Comparison of the estimated Gutenberg–Richter b-values from recorded earthquakes in major seismotectonic provinces of Iran with those obtained from previous works done with other methods show that estimation of b-value using the method without levels of completeness (M_C) has been successful. This method can reduce time duration for evaluation of seismicity parameters.

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