An efficient algorithm for large-scale joint inversion of gravity and magnetic data

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ABSTRACT

An efficient algorithm for large-scale joint inversion of gravity and magnetic data sets using Gramian coupling constraint is developed. The global objective function is formulated in the space of weighted parameters, but the Gramian constraint is implemented in the original unweighted space. This provides more similarity between reconstructed models. It is assumed that measured data are obtained on a uniform grid. Then, the sensitivity matrices exhibit a block Toeplitz Toeplitz block (BTTB) structure for each depth layer of the model domain, and both forward and transpose operations with the matrices can be implemented efficiently using two dimensional fast Fourier transforms (FFT). The regularized reweighted conjugate gradient (RRCG) algorithm, which relies only on matrix vector multiplication, is then used to minimize the objective function. Application of the RRCG algorithm in conjunction with the BTTB structure of the sensitivity matrices leads to a very fast methodology for large-scale joint inversion of gravity and magnetic data sets. Numerical simulations and real data application demonstrate the efficiency of the presented joint inversion algorithm.

Keywords: gravity, magnetic, joint inversion, large-scale, Gramian constraint, BTTB structure

INTRODUCTION

Different geophysical data sets acquired in a given survey area provide information about different physical properties of the subsurface targets. The joint inversion of such multiple data sets can provide a more reliable model than a model which is produced by only one single data set. For the joint inversion algorithms, several coupling measures have been developed for the linkage between different model parameters (Lelièvre et al., 2012). Recently, the Gramian determinant constraint has been used extensively for the joint inversion of geophysical data sets (Zhdanov, 2015). The Gramian coupling does not require any a priori knowledge of specific relationship between model parameters. Indeed, with minimization of the Gramian constraint, during the inversion process, we enforce a linear relationship between different model parameters and/or their attributes. On the other hand, there are two major computational obstacles for the joint inversion of potential field data sets; the high storage requirements and the high computational costs. For a uniform grid of data points, the model sensitivity matrices have a BTTB structure for each depth layer of the model. Then, all forward and transpose matrix operations can be implemented using 2D FFT (Vogel, 2002). In this case, rather than storing the dense sensitivity matrices, it is sufficient to store a limited number of vectors. Furthermore, here, I use the regularized reweighted conjugate gradient (RRCG) algorithm, in conjunction with the BTTB structure of the sensitivity matrices, to minimize the global objective function. Within the framework of the RRCG algorithm matrix inversions are avoided. This makes it feasible to solve for large scale problems with respect to both computational costs and memory demands. To generate compact and sparse solution, here, I use L_1-norm of model parameter as stabilizer. The presented algorithm is tested on synthetic model and applied for a real data case obtained over an area in northwest of Mesoproterozoic St. Francois Terrane, southeast of Missouri, USA.

METHODOLOGY

The subsurface is divided into a set of rectangular prisms, in which the prisms are kept fixed during the inversion, and unknown values of the physical properties of the prisms, density and magnetic susceptibility, are the model parameters which should be estimated. Here, we assume that data points are obtained on a uniform grid. Further, we suppose that there is no remanent magnetization. The forward operators are given by

$$\mathbf{d}_{\text{obs}}^{(i)} = \mathrm{A}^{(i)}\mathbf{m}^{(i)}, \qquad i = 1, 2. \tag{1}$$

where the vectors $\mathbf{d}_{\text{obs}}^{(i)} \in \mathbb{R}^m$ are the observed gravity and magnetic data. Here I use i=1 for gravity and i=2 for magnetic, respectively. The matrices $\mathrm{A}^{(i)} \in \mathbb{R}^{m\times n}$ are the known sensitivity matrices which have BTTB structure for each depth layer. The goal is to find geologically acceptable models $\mathbf{m}^{(i)} \in \mathbb{R}^n$ that predict observed data at noise levels via a simultaneous joint inversion. The global objective function which should be minimized is given by (Vatankhah et al., 2022)

$$\mathrm{P}^{(\alpha,\lambda)}(\mathbf{m}^{(1)},\mathbf{m}^{(2)}) = \sum_{i=1}^{2}\left\|\mathrm{W}_{\mathrm{d}}^{(i)}(\mathrm{A}^{(i)}\mathbf{m}^{(i)} - \mathbf{d}_{\text{obs}}^{(i)})\right\|_2^2 + \sum_{i=1}^{2}\alpha^{(i)}\left\|\mathrm{W}^{(i)}(\mathbf{m}^{(i)} - \mathbf{m}_{\text{apr}}^{(i)})\right\|_2^2 + \lambda S_G(\mathbf{m}^{(1)},\mathbf{m}^{(2)}). \qquad (2)$$

Where the weighting matrix on model parameters and Gramian constraint are given by

$$\mathrm{W}^{(i)} = \mathrm{W}_{\text{depth}}^{(i)}\mathrm{W}_{\text{hard}}^{(i)}\mathrm{W}_{\mathrm{L}_1}^{(i)}, \quad \mathrm{W}_{\mathrm{L}_1}^{(i)} = \mathrm{diag}\left(1\Big/((\mathbf{m}^{(i)} - \mathbf{m}_{\text{apr}}^{(i)})^2 + \varepsilon^2)^{1/4}\right), \quad S_G(\mathbf{m}^{(1)},\mathbf{m}^{(2)}) = \begin{vmatrix} (\mathbf{m}^{(1)},\mathbf{m}^{(1)}) & (\mathbf{m}^{(1)},\mathbf{m}^{(2)}) \\ (\mathbf{m}^{(2)},\mathbf{m}^{(1)}) & (\mathbf{m}^{(2)},\mathbf{m}^{(2)}) \end{vmatrix}$$

The minimization of eq. (2), then, enforces a linear correlation between model parameters, $\mathbf{m}^{(1)} = \gamma\mathbf{m}^{(2)}$ (γ is a scalar), and no a priori knowledge is required.

The objective function given in eq. (2) can be reformulated using a space of weighted parameters as

$$\mathrm{P}^{(\alpha,\lambda)}(\tilde{\mathbf{m}}^{(1)},\tilde{\mathbf{m}}^{(2)}) = \sum_{i=1}^{2}\left\|\tilde{\mathrm{A}}^{(i)}\tilde{\mathbf{m}}^{(i)} - \tilde{\mathbf{d}}_{\text{obs}}^{(i)}\right\|_2^2 + \sum_{i=1}^{2}\alpha^{(i)}\left\|\tilde{\mathbf{m}}^{(i)} - \tilde{\mathbf{m}}_{\text{apr}}^{(i)}\right\|_2^2 + \lambda S_G(\mathbf{m}^{(1)},\mathbf{m}^{(2)}). \qquad (3)$$

Where $\tilde{\mathrm{A}}^{(i)} = \mathrm{W}_{\mathrm{d}}^{(i)}\mathrm{A}^{(i)}(\mathrm{W}^{(i)})^{-1}$, $\tilde{\mathbf{d}}_{\text{obs}}^{(i)} = \mathrm{W}_{\mathrm{d}}^{(i)}\mathbf{d}_{\text{obs}}^{(i)}$, $\tilde{\mathbf{m}}^{(i)} = \mathrm{W}^{(i)}\mathbf{m}^{(i)}$, $\tilde{\mathbf{m}}_{\text{apr}}^{(i)} = \mathrm{W}^{(i)}\mathbf{m}_{\text{apr}}^{(i)}$.

Here data misfit and stabilizer terms are given in the space of weighted model parameter, but the Gramian constraint is given in the original space. This strategy provides a more robust approach for selection of the regularization and weighting parameters, while, simultaneously, provides the correlation between the original model parameters and not their weighted forms (see Vatankhah et al. (2022) for more details).

For the minimization of (3), the regularized re-weighted conjugate gradient algorithm is used

$$\begin{aligned}
&\bar{\tilde{\mathbf{r}}}_k = \bar{\tilde{\mathrm{A}}}_k\bar{\tilde{\mathbf{m}}}_k - \bar{\tilde{\mathbf{d}}}_{\text{obs}}, \\
&(l_G^{(1)})_k = \left\|\mathbf{m}_k^{(2)} - \right\|_2^2 \mathbf{m}_k^{(1)} - (\mathbf{m}_k^{(1)},\mathbf{m}_k^{(2)})\mathbf{m}_k^{(2)}, \qquad (l_G^{(2)})_k = \left\|\mathbf{m}_k^{(1)} - \right\|_2^2 \mathbf{m}_k^{(2)} - (\mathbf{m}_k^{(1)},\mathbf{m}_k^{(2)})\mathbf{m}_k^{(1)}, \\
&\bar{l}_k = \bar{\tilde{\mathrm{A}}}_k^{\mathrm{T}}\bar{\tilde{\mathbf{r}}}_k + \bar{\alpha}_k(\bar{\tilde{\mathbf{m}}}_k - \bar{\tilde{\mathbf{m}}}_{k-1}) + \bar{\lambda}(\bar{l}_G)_k, \\
&\bar{\mathrm{p}}_k = \bar{l}_k + \frac{\left\|\bar{l}_k\right\|_2^2}{\left\|\bar{l}_{k-1}\right\|_2^2}\bar{\mathrm{p}}_{k-1}, \qquad \bar{\mathrm{p}}_0 = \bar{l}_0, \qquad (3) \\
&s_k = (\bar{\mathrm{p}}_k,\bar{l}_k)\Big/\left[\left\|\bar{\tilde{\mathrm{A}}}_k\bar{\mathrm{p}}_k\right\|_2^2 + \bar{\alpha}_k\left\|\bar{\mathrm{p}}_k\right\|_2^2 + \bar{\lambda}\left\|\bar{\mathrm{p}}_k\right\|_2^2\right], \\
&\bar{\tilde{\mathbf{m}}}_{k+1} = \bar{\tilde{\mathbf{m}}}_k - s_k\bar{\mathrm{p}}_k, \\
&\bar{\mathbf{m}}_{k+1}^{(i)} = (\mathrm{W}_k^{(i)})^{-1}\bar{\tilde{\mathbf{m}}}_{k+1}^{(i)}.
\end{aligned}$$

Further, at each iteration of RRCG algorithm, lower and upper bounds are also imposed on obtained model parameters. From the steps of the RRCG algorithm it is clear that only forward and transpose matrix-vector multiplications are required (there is no need to matrix inversion). All such operations can proceed by using the BTTB structure of the sensitivity matrices and 2D FFT, as described in detail in Hogue et al. (2020) and Vatankhah et al. (2022), and I do not repeat here. Hence, the RRCG algorithm provides a very fast methodology for minimizing the global objective function (3), which also requires minimal memory storage. Then, large-scale joint inversion problems can be implemented very fast. In the presented algorithm, all weighting parameters $\alpha^{(i)}$ and $\lambda^{(i)}$ have to be chosen carefully. I use a simple but effective strategy to determine these parameters. At first iteration, large values for $\alpha^{(i)}$ are selected. At next iterations, these parameters are reduced slowly. For the parameters $\lambda^{(i)}$, I use fixed values during iterations.

SYNTHETIC MODEL

The algorithm is tested on a large-scale complicated model that consists of five bodies with different shapes, sizes, and depths. A 3D iso-surface of the model is illustrated in Figure 1(a). Further, three depth section of the density distribution of the model are shown in Figure 2. The gravity and magnetic data sets are generated on the surface over the 150 ×100 uniform grid with 100 m grid spacing. The noise-contaminated data sets are illustrated in Figures 1(b) and 1(c). To perform the inversion, the subsurface is divided into 150×100×10=150000 prisms of size 100 m in each dimension. The computation is performed on a standard laptop computer with Intel(R) Core(TM) i7-10750H CPU 2:6 GHz processor and 16 GB RAM. After 96 iterations, and about 4872 s, the convergence criteria are satisfied and inversion terminates. Figures 3 and 4 are illustrated three depth sections of the reconstructed density and magnetic susceptibility models. The models are sparse, geophysically acceptable, similar, and consistent with true models.

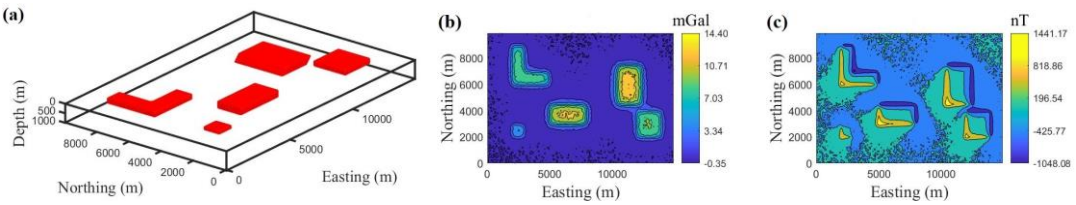

Figure 1. (a) Synthetic model consisting of five different bodies. (b) and (c) are the noise-contaminated gravity and magnetic data produced by the model.

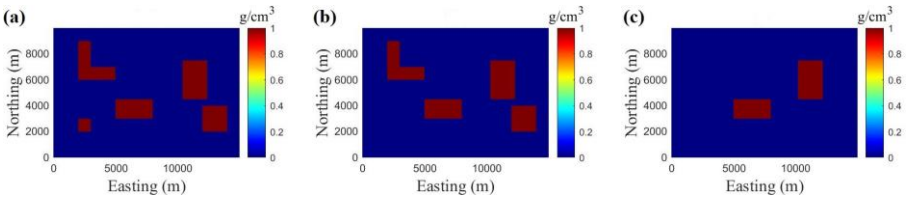

Figure 2. Three depth sections of the original density distribution for the model shown in Figure 1(a). At depth (a) 200m, (b) 300 m, and (c) 400.

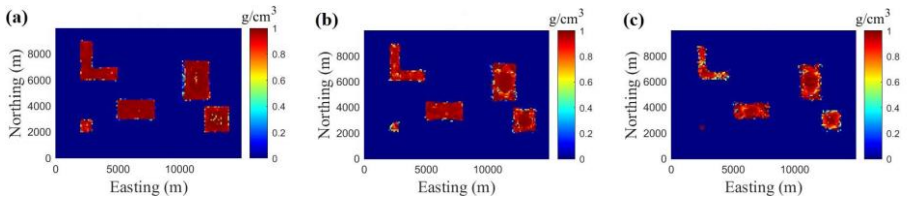

Figure 3. Three depth sections of the reconstructed density model. At depth (a) 200m, (b) 300 m, and (c) 400.

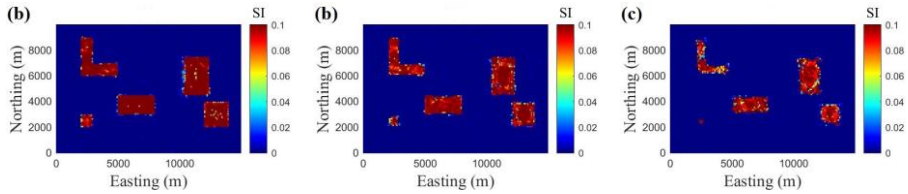

Figure 4. Three depth sections of the reconstructed magnetic susceptibility model. At depth (a) 200m, (b) 300 m, and (c) 400.

REAL DATA

The joint inversion algorithm is applied on gravity and magnetic data obtained over the northwestern portion of the Mesoproterozoic St. Francois Terrane in the southeast Missouri, USA. Major targets in the survey area are the magnetic, magnetic breccia and hematite ore bodies. These igneous rocks are covered by 1-2 km of Paleozoic limestones and sandstones. Figure 5 illustrates the residual gravity and magnetic data over the survey area. Here, there are 21608 gridded data points. The Pea Ridge (PR) magnetic deposit, Figure 5, is the only deposit that also contains significant rare earth elements (REE).

To perform the inversion, the subsurface was divided into 146×148×20=432160 prisms. The inversion algorithm terminates after 186 iterations with a run time of approximately 20 hours. Three cross-sections of the reconstructed density and magnetic susceptibility models over the major targets are illustrated in Figures 6 and 7.

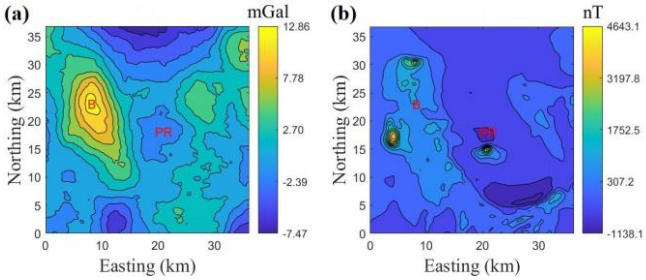

Figure 5. (a) Residual gravity, and (b) residual magnetic anomalies over the survey area. The locations of major Fe oxide deposits including Bourbon (B) and Pea Ridge (PR) are shown.

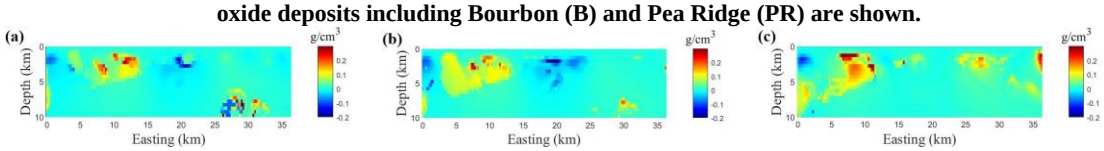

Figure 6. Cross-sections of the reconstructed density model. The sections are at (a) 16 km Northing (over PR), (b) 18 km Northing, and (c) 24 km Northing.

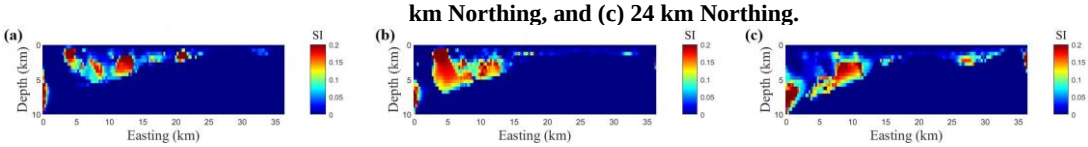

Figure 7. Cross-sections of the reconstructed magnetic susceptibility model. The sections are at (a) 16 km Northing (over PR), (b) 18 km Northing, and (c) 24 km Northing.

CONCLUSION(S)

An algorithm for large-scale focusing joint inversion of gravity and magnetic data using Gramian coupling constraint has been presented. The objective function is formulated in the space of weighted parameters, but the Gramian constraint is implemented in the original space. This provides more similarity between the reconstructed models. To solve large-scale problems, the BTTB structure of the sensitivity matrices and 2D FFT in conjunction with RRCG algorithm was used. Then, the presented algorithm is efficient for both memory storage and computational cost. The algorithm was tested on a large complicated synthetic model, and then applied on real data set obtained over an area in the southeast Missouri, USA.

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